Wall Region Heat and Mass Transfer With Newtonian and Viscoelastic Fluids in Turbulent Flow

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Heat and mass transfer models that represent the experimental range of Prandtl and Schmidt numbers with turbulent flow include empirical wall region models (Churchill, 1977; Hughmark, 1975*b*). Shaw and Hanratty (1977) reported experimental data to rule out the plausible limiting expressions for eddy diffusivity close to a wall. Lau, Lee, and Hanratty (1977) interpret additional experimental data to indicate that the region $y^+ <$ about 4 cannot be represented by a pseudo steady state assumption. Hughmark (1972, 1973) analyzed experimental data to indicate that the core and transition regions can be represented by pseudo steady state models. This communication utilizes the models for the core and transition region with a range of experimental heat and mass transfer data to estimate the boundary layer contribution at the wall. Data for viscoelastic fluids in turbulent flow are also considered with respect to these models.

NEWTONIAN FLUIDS

The prior work related to steady state models for the core and transition region show that eddy transport in the core of a circular pipe is represented by

$$k^+_{EC} = 2\sqrt{f/2} \tag{1}$$

and for the transition region by

$$k^+_{ET} = 0.0615\, N_{Sc}^{-1/2} \tag{2}$$

Equation (2) represents the penetration model with the contact time consistent with the wall frequency relationship

$$\frac{u^{*2}t}{\nu} = 338 \tag{3}$$

Also, in the prior work, eddy diffusivity for heat and mass for the transition region was correlated by

$$\frac{\epsilon}{\nu} = 0.0094\, y^{+2} \tag{4}$$

Integration of the eddy diffusivity with the upper limit as $y^+ = 34.6$ to represent the boundary with the turbulent core shows that the lower limit is $y^+ = 5.5$ to correspond to the eddy transport of Equation (2). This is in reasonable agreement with $y^+ =$ about 4 value reported by Lau et al. as the limit of the wall boundary layer that cannot be represented by a pseudo steady state assumption.

The sum of the eddy diffusivity from Equation (4) and the molecular diffusivity for heat or mass can be integrated from $y^+ = 5.5$ to $y^+ = 34.6$ to provide a transport coefficient. For heat transport this is

$$k_T^+ = \left(\frac{0.0094}{N_{Pr}}\right)^{1/2} \frac{N_{Pr}^{-1/2}}{a\tan[34.6(0.0094\, N_{Pr})^{1/2}] - a\tan[5.5(0.0094\, N_{Pr})^{1/2}]} \tag{5}$$

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If molecular transport is assumed negligible for the core, Equations (1) and (5) can be utilized with the resistance model

$$\frac{1}{k^+} = \frac{1}{k^+_B} + \frac{1}{k^+_T} + \frac{1}{k^+_{EC}} \tag{6}$$

to estimate the wall boundary layer coefficient. Figure 1 shows the data of Debrule and Sabersky (1974), Friend and Metzner (1958), Harriott and Hamilton (1965), Malina and Sparrow (1964), and Shaw and Hanratty (1977). The correlated data for $N_{Re} = 10\,000$ of Bernardo, Everett, and Eian as listed by Friend and Metzner (1958) is also shown, as this fills a range not covered by the other data. The data represent a Prandtl or Schmidt number range from 3 to 97 500. At $N_{Pr} = 3$ and $N_{Re} = 10\,000$, about 15% of the transport resistance is in the wall boundary layer, so calculated values for $N_{Pr} < 3$ are not shown because these calculated values approach the error in the data. The data of Figure 1 are correlated by

$$k_B^+ = 0.547(N_{Pr} \text{ or } N_{Sc})^{-0.88} \tag{7}$$

which is consistent with the Shaw and Hanratty observation that the observed wall region value rules out the plausible limiting expressions for eddy diffusivity close to a wall. Figure 1 is interesting in that the Harriott and Hamilton data are correlated by Equation (7) for $N_{Sc} <$ 3 000 but show significantly higher coefficients at the higher Schmidt numbers. The Harriott and Hamilton data were obtained by the wall solution method and the Shaw and Hanratty data by the electrochemical method.

VISCOELASTIC FLUIDS

Hughmark (1975*a*) showed that the wall region frequency for a viscoelastic fluid in turbulent flow could be related to the drag reduction of the fluid with a flow behavior index of unity

$$\frac{f_N}{f_E} = 0.0206\, \alpha^{2/3} \tag{8}$$

which corresponds to

$$k^+_{ET} = 1.13\, \alpha^{-1/2}\, N_{Sc}^{-1/2} \tag{9}$$

for the penetration model with this wall region frequency. The dimensionless variable α is a function of the Deborah number as shown by the reference. Similarly, the eddy diffusivity for the transition region is

$$\frac{\epsilon}{\nu} = 3.17/\alpha\, y^{+2} \tag{10}$$

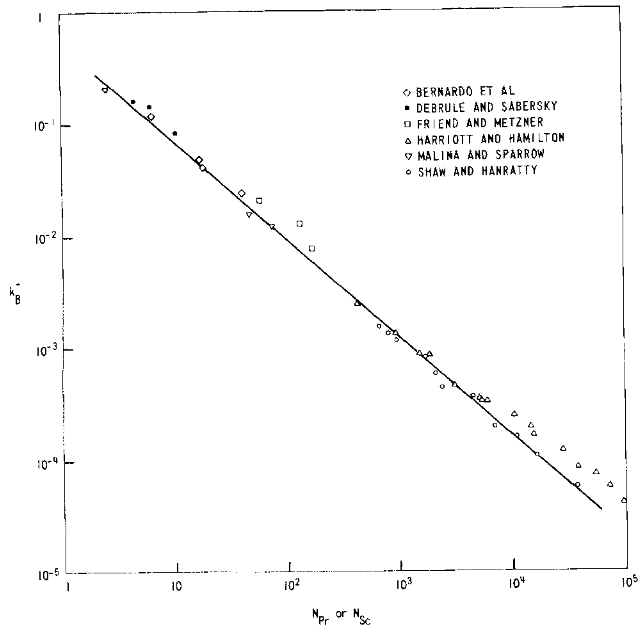

Fig. 1. Wall boundary layer coefficients.

McConaghy and Hanratty (1977) reported mass transfer data with Schmidt numbers of about 700 for a Newtonian fluid and the range of 946 to 1 264 with drag reducing additive to the fluid. The predicted values from Equations (1), (2), (6), and (7) show an average absolute deviation of 4.2% with the Newtonian fluid data. Predicted values from Equations (1), (6), (7), and (9) show an average absolute deviation of 7.2% with the viscoelastic fluid data. These results indicate that the wall boundary layer correlation for Newtonian fluids is also applicable for viscoelastic fluids in turbulent flow.

The eddy diffusivity for the transition region and the molecular diffusivity can be integrated for viscoelastic fluids as for Newtonian fluids. Viscoelastic fluids also show increased boundary layer thicknesses. One assumption for this is that the ratio of the eddy diffusivity to the molecular diffusivity remains constant at the boundaries of the transition region. This has the effect of increasing the thickness of the wall boundary layer and the transition region. The resulting equation for heat transfer is

$$k_T{}^+ = \left(\frac{3.17}{\alpha N_{Pr}}\right)^{1/3} \frac{N_{Pr}{}^{-1/3}}{\alpha \tan[34.6(0.0094\, N_{Pr})^{1/2}] - \alpha \tan[5.5(0.0094\, N_{Pr})^{1/2}]} \quad (11)$$

Gupta, Metzner, and Hartnett (1967) reported heat transfer data for water and 500 ppm ET-597 in water. Comparison of predicted with experimental values for water with the Newtonian model and for the drag reducing fluid with Equations (1), (6), (7), and (11) show average absolute deviations of 14.6 and 11%, respectively. Smith, Keuroghlian, Virk, and Merrill (1969) reported heat transfer data for water and 10 ppm N 3 000 in water. A similar comparison of predicted with experimental values shows average absolute deviations of 5.2% for water and 6.7% for the N 3 000 solution. Thus, the heat transfer data with Prandtl numbers in the range of 6 to 11 also indicate that the wall boundary layer correlation for Newtonian fluids is applicable for viscoelastic fluids in turbulent flow.

CONCLUSIONS

An estimation of the wall boundary layer response over a wide range of Prandtl and Schmidt numbers with turbulent flow confirms that the plausible limiting expressions for eddy diffusivity close to a wall do not apply. The estimated wall boundary layer response also appears to apply to viscoelastic fluids with turbulent flow.

NOTATION

f = friction factor
k^+ = dimensionless transfer coefficient
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
N_{Sc} = Schmidt number
t = eddy contact time
u^* = shear velocity
y^+ = dimensionless distance from wall
α = defined by Equation (8)
ϵ = eddy diffusivity
ν = kinematic viscosity

Subscripts

B = wall boundary layer
E = viscoelastic
EC = eddy core
ET = eddy transition region
N = Newtonian
T = transition region

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